COLLEGE
Tanta Engineering

إعدادي 2020

الهندسة التحليلية

المعادلة المزدوجة لمنصف الزاوية

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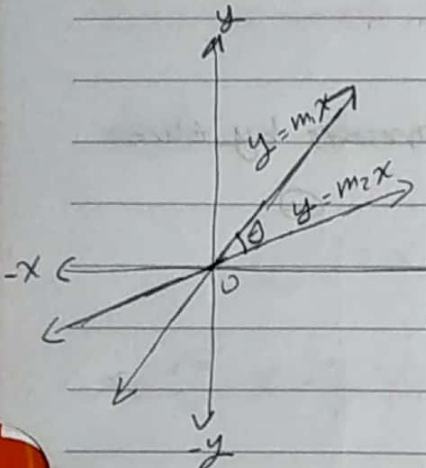
2nd

$$ax^2 + 2hxy + by^2 = 0 \quad \text{--- } (*)$$



$$y = m_1 x \quad , \quad y = m_2 x$$

$$m_1 = \frac{-h + \sqrt{h^2 - ab}}{b} \quad , \quad m_2 = \frac{-h - \sqrt{h^2 - ab}}{b}$$

$$m_1 + m_2 = \frac{-2h}{b} \quad , \quad m_1 m_2 = \frac{a}{b}$$

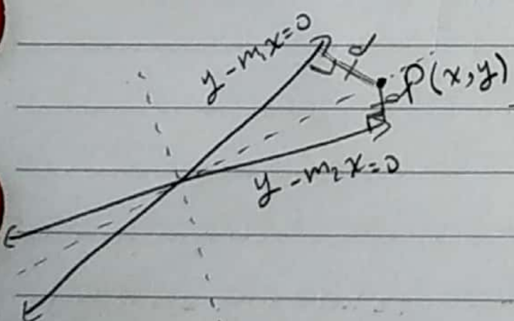
$$\tan\theta = \frac{m_2 - m_1}{1 + m_1 m_2}$$

$$= \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2}$$

$$= \frac{\sqrt{\frac{4h^2}{b^2} - 4\frac{a}{b}}}{1 + \frac{a}{b}}$$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

Double eqn of bisectors of θ



$$\frac{y - m_1 x}{\sqrt{1 + m_1^2}} = \pm \frac{y - m_2 x}{\sqrt{1 + m_2^2}}$$

$$\left(\frac{y - m_1 x}{\sqrt{1 + m_1^2}} - \frac{y - m_2 x}{\sqrt{1 + m_2^2}}\right)\left(\frac{y - m_1 x}{\sqrt{1 + m_1^2}} + \frac{y - m_2 x}{\sqrt{1 + m_2^2}}\right) = 0$$

d (x_1, y_1)

$ax + by + c$

$$d = \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}$$

$$\frac{(y - m_1 x)^2}{1 + m_1^2} - \frac{(y - m_2 x)^2}{1 + m_2^2} = 0$$

$$(1 + m_2^2)(y^2 - 2m_1 xy + m_1^2 x^2) - (1 + m_1^2)(y^2 - 2m_2 xy + m_2^2 x^2) = 0$$

$$x^2(m_1^2 + \cancel{m_1^2 m_2^2} - \cancel{m_1^2 m_2^2} - m_2^2) + y^2(\cancel{1} + m_2^2 - \cancel{1} - m_1^2)$$

$$-2xy(m_1 + m_1 m_2^2 - m_2 - m_2 m_1^2) = 0$$

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$$(x^2-y^2)(m_1\cancel{-m_2})(m_1+m_2)-2xy\left((m_1\cancel{-m_2})(1-m_1m_2)\right)=0$$

$$\frac{\cancel{-2}h}{b}(x^2-y^2)\cancel{+2}xy\left(1-\frac{a}{b}\right)=0$$

$$\frac{h}{b}(x^2-y^2)+xy\left(1-\frac{a}{b}\right)=0$$

$$h(x^2-y^2)+xy(b-a)=0$$

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

Ex: find θ and the double bisector eqn of the last Examble.

$$y^2-7xy+10x^2=0$$

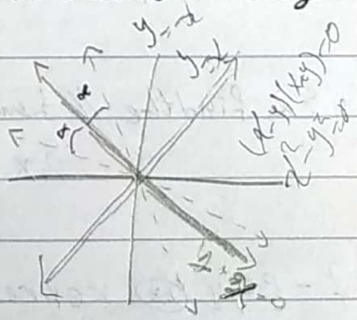
$a=10$, $h=\frac{-7}{2}$, $b=1$

$$\tan\theta=\frac{2\sqrt{\frac{49}{4}-10}}{10+1}=\frac{3}{11} \quad \therefore\ \theta=\tan^{-1}\left(\frac{3}{11}\right)=15{,}255^\circ$$

$$\frac{x^2-y^2}{10-1}=\frac{xy}{-7/2}$$

Ex: find the double eqn of line pair passing the o and make an angle α with the line $x+y=0$ $\quad \theta=2\alpha$

$$ax^2+2hxy+by^2=0$$



D. of bisectors is given by: $x^2-y^2=0 \to (1)$

But $\left(x^2-y^2=\frac{a-b}{h}\cdot xy\right) \to (2)$

Comparing (1), (2): $\frac{a-b}{h}=0 \Rightarrow a=b$

$\tan 2\alpha=\frac{\cancel{2}\sqrt{h^2-a^2}}{\cancel{2}a}$ $\quad \therefore\ a^2\sec^2 2\alpha=h^2 \quad \therefore\ h=\pm a\sec 2\alpha$

$$\therefore\ \cancel{a}x^2\pm 2\cancel{a}\sec 2\alpha\, xy+\cancel{a}y^2=0$$

$$x^2\pm 2\sec 2\alpha\, xy+y^2=0$$

Condition for the eqn $ax^2+2hxy+by^2+2fx+2gy+C=0$ (**) to represent two line:

$$by^2+2(hx+g)y+ax^2+2fx+C=0$$

$$y=\frac{-\cancel{2}hx+y\pm\sqrt{\cancel{4}(hx+g)^2-\cancel{4}[abx^2+2fbx+bc]}}{\cancel{2}b}$$

discreminate $=(hx+g)^2-[abx^2-2fbx+bc]\equiv$ Perfect square

$$x^2(h^2-ab)+2x(hg-bf)+(g^2-bc)=\text{perfect square}$$

$$Disc_2=0=4((hg)-bf)^2-4(h^2-ab)(g^2-bc)=0$$

OR:

$$\Delta=\begin{vmatrix} a & h & f \\ h & b & g \\ f & g & c \end{vmatrix}=0$$

Notes:

1- The two lines $a_1x+b_1y+C_1=0$, $a_2x+b_2x+C_2=0$ are
a) // if $\frac{a_1}{a_2}=\frac{b_1}{b_2}$ b) the same if $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$

Ex: find the // line to $2x+3y+5=0$ and pasing the $(1,2)$
$2x+3y+C=0$ and sub to get C

2- eqn (**) represents two lines // to eqn (*)

Ex. find the two lins, θ, the two bisectors for the eqn

$$y^2 - 7xy + 10x + y - 12 = 0$$

$$(y-2x)(y-5x) = 0$$

$$y - 2x = 0 \quad , \quad y - 5x = 0$$

$$\cancel{y - 2x + \alpha = 0 \quad , \quad y - 2x + \beta = 0}$$

$$(y - 2x + \alpha)(y - 5x + \beta) = 0$$

$$C.O.x, \quad 7 = -2\beta - 5\alpha \longrightarrow ①$$

$$C.O.y: \quad 1 = \beta + \alpha \longrightarrow ②$$

$$9 = -3\alpha \quad \therefore \alpha = -3 \quad , \quad \beta = 4$$

$$C = \alpha\beta$$

$$-12 = -3 \times 4$$

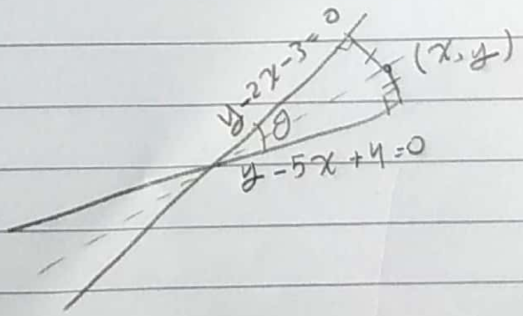
$$-12 = -12 \quad \checkmark$$

$$y - 2x - 3 = 0 \quad , \quad y - 5x + 4 = 0 \quad \#$$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a + b}$$

$$\theta = 15,255^\circ \#$$

$$\frac{y - 2x - 3}{\sqrt{1+4}} = \pm \frac{y - 5x + 4}{\sqrt{1 + 25}}$$



$$\frac{y - 2x - 3}{\sqrt{5}} = \pm \frac{y - 5x + 4}{\sqrt{26}}$$

$$\sqrt{26}\,(y - 2x - 3) = \sqrt{5}\,(y - 5x + 4)$$

$$\sqrt{26}\,(y - 2x - 3) = -\sqrt{5}\,(y - 5x + 4) \quad \#$$